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Spin–charge separation in strongly correlated electronic systems

Naoto Nagaosa

Department of Applied Physics, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

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Abstract. The spin–charge separation (SCS) in 1D and 2D are discussed from the viewpoint of gauge theory. For 1D I discuss the angle-resolved photoemission spectra (ARPES), which show clear evidence for the spin–charge separation. For 2D the underdoped cuprates are discussed, where the three classes of electronic state, i.e., the Néel state (N), the valence-bond solid (VBS) state and the resonating-valence-bond (RVB) state, are relevant. These states can be understood in terms of the competition between (i) magnetic ordering versus singlet formation and (ii) confinement versus deconfinement of the gauge field. It is fairly easy to understand the former, (i), but the latter, (ii), is more subtle and has not yet been established. I argue that the deconfining phase, i.e., the RVB state, is a *new state of matter* with SCS, and is realized when the sheet resistance R_{2D} is less than a critical value of the order of the quantum resistance $R_Q = h/4e^2$. This condition is equivalent to that for superconductivity in the Josephson network model. The anomalous Kondo effect due to the non-magnetic impurities doped into the system reflects the non-Fermi-liquid nature of the host electronic state, and hence is the most promising experimental evidence for this new state of matter. We put special emphasis on the residual resistivity, and propose that its value provides a clear test for SCS.

1. Introduction

Spin–charge separation (SCS) is one of the key issues in the physics of strongly correlated electronic systems. The non-Fermi-liquid state in 1D has been studied for a long time, since the seminal work by Tomonaga [1] and, later, by Luttinger [2]. In 1D, the low-energy excitations are exhausted by the collective ones because the Fermi surface consists of just two points at $k = \pm k_F$. The collective modes are more sensitive to the interactions than the individual ones, and it is true even in 3D that the spin and charge collective excitations are separated and decoupled. The Tomonaga–Luttinger (TL) liquid is a very general concept in 1D systems where the large quantum fluctuation destroys the long-range ordering (LRO) even at $T = 0$, and hence the quantum liquid is realized. The AF Heisenberg spin chain is a typical example of this, for which there is no antiferromagnetic long-range order (AFLRO). When the spin quantum number S is half an odd integer, the excitation spectrum is gapless, while it has a gap for integer S [3]. For the integer-spin Heisenberg chain and the two-leg-ladder Heisenberg system [4], the spin gap can be understood in terms of the valence-bond solid (VBS) picture, i.e., the solid state of the singlet pair. In this sense, it is similar to the spin–Peierls system, where the lattice dimerization is accompanied by a singlet solid. In the field theoretical formulation, the difference between half-odd-integer and integer spin comes from the topological property, i.e., the Berry phase, in the non-linear sigma model [3]. Hence the gauge field is highly relevant to the whole issue.

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The discovery of high-T_c cuprates in 1986 triggered theoretical studies searching for the new state of matter induced by the strong electron correlation in quasi-2D electronic systems. The most novel of these is the RVB (resonating-valence-bond) state first proposed by Anderson at the early stages of high-T_c research [5]. Due to the strong quantum fluctuation in the antiferromagnetic spin system, the Néel order is sometimes destroyed and the quantum spin liquid is realized as in 1D systems. However, the situation is less clear in 2D. Although many theoretical studies are guided by the 1D results, the relationship between 1D and 2D is not revealed sufficiently at present. This issue is becoming of even keener interest due to the discovery of superconductivity in the two-leg-ladder compound stemming from the spin-gap state in the undoped material. Thus one of the most important theoretical issues at present is how to characterize and identify the states in the strongly correlated systems.

In this paper, we report on some of the recent progress on the SCS in 1D and 2D strongly correlated systems. In section 2 we give a brief review of the gauge theory of SCS. In section 3 the angle-resolved photoemission spectrum (ARPES) is studied from the gauge theoretical viewpoint. In sections 4 and 5 we study the underdoped 2D high-T_c cuprates in terms of the recently developed SU(2) gauge theory. There are the two criteria used to classify the states of matter. One is the competition between the magnetic ordering and the singlet formation. The other criterion is more subtle and is not yet established. That is the confinement versus deconfinement of the gauge field appearing in the lattice gauge theoretical formulation of the strongly correlated electronic systems. We propose in section 5 that the anomalous Kondo effect is important evidence for the SCS; this is explained also in terms of the SU(2) theory.

2. Gauge theory of the spin–charge-separated system

A formulation for the spin–charge separation is developed in terms of the slave-boson formalism combined with the gauge theory. We start with the t–J model which is considered to be a model for high-T_c cuprates [5]. The Hamiltonian is

$$H = \sum_{(ij)} \left[J\left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j - \frac{1}{4} n_i n_j \right) - t\,(c^\dagger_{\alpha i} c_{\alpha j} + \mathrm{HC}) \right] \tag{1}$$

where the electron operator $c^\dagger_{i\sigma}$ creates an electron with spin σ on site i, and the spin operator $\boldsymbol{S}_i$ is given by

$$\boldsymbol{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta} \tag{2}$$

where the $\boldsymbol{\sigma}$ are the Pauli matrices. The most important feature of the t–J model is the constraint that two electrons cannot occupy the same site because of the strong repulsive interaction U. In the slave-boson method [5–7], this constraint is taken into account by representing the electron operator by a product of spinon (fermion) operators, $f^\dagger_{i\sigma}$, and holon (boson) operators, b_i, as follows:

$$c^\dagger_{i\sigma} = f^\dagger_{i\sigma} b_i. \tag{3}$$

This means that the creation of an electron corresponds to the annihilation of a vacancy (holon) and the creation of a spin (spinon). The physical states satisfy the local constraint

$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + b^\dagger_i b_i = 1 \tag{4}$$

corresponding to the three possible states for each site. Then the partition function Z of the t–J model is represented in terms of the functional integral as

$$Z = \int \mathrm{D}\psi\ \mathrm{D}\psi^\dagger\ \mathrm{D}b\ \mathrm{D}b^*\ \mathrm{D}U\ \mathrm{D}a_0\ \exp\left(-\int_0^\beta L\right) \tag{5}$$

with the Lagrangian L being given by

$$\begin{aligned} L = {} & \frac{\tilde{J}}{2}\sum_{\langle ij\rangle} \mathrm{Tr}\big[U_{ij}^\dagger U_{ij}\big] + \frac{1}{2}\sum_{ij\alpha} \psi_{i\alpha}^\dagger\big[(\partial_\tau + a_0\tau_3)\delta_{ij} + \tilde{J}U_{ij}\big]\psi_{j\alpha} \\ & + \sum_i b_i^\dagger(\partial_\tau - \mu + a_0)b_i - t\sum_{ij}\chi_{ij} b_j^\dagger b_i \\ = {} & L_F + L_B. \end{aligned} \tag{6}$$

The first line is the Lagrangian L_F for the fermions while the second line is the contribution L_B from the doped holes. We have introduced the Stratonovich–Hubbard field U_{ij} to decompose the interactions, and the Lagrange multiplier field a_0 to impose the constraint equation (4). Here the matrix U_{ij} has the spinon pairing order parameter Δ_{ij} and the hopping order parameter χ_{ij} as matrix elements, i.e.,

$$U_{ij} = \begin{bmatrix} -\chi_{ij}^* & \Delta_{ij} \\ \Delta_{ij}^* & \chi_{ij} \end{bmatrix}. \tag{7}$$

The spinor $\psi_{i\alpha}$ is given by

$$\psi_{1i} = \begin{pmatrix} f_{1i} \\ f_{2i}^\dagger \end{pmatrix} \qquad \psi_{2i} = \begin{pmatrix} f_{2i} \\ -f_{1i}^\dagger \end{pmatrix}. \tag{8}$$

The mean-field theory of the spin–charge-separated state corresponds to the saddle-point approximation to the functional integral over $\chi_{ij}, \Delta_{ij}, a_0$.

The gauge theory is derived by considering the fluctuation around the saddle point. In particular, the gauge invariance with respect to the local gauge transformation

$$\begin{aligned} f_{i\sigma} &\to f_{i\sigma}\mathrm{e}^{\mathrm{i}\theta_i} \\ b_i &\to b_{i\sigma}\mathrm{e}^{\mathrm{i}\theta_i} \end{aligned} \tag{9}$$

must be respected. This gauge invariance is closely connected to the constraint equation (4), which has been treated in an averaged way in the mean-field theory. To recover the gauge invariance we need a gauge field which transforms according to equation (9) as

$$a_{ij} \to a_{ij} + \theta_i - \theta_j. \tag{10}$$

The phase of the order parameter χ_{ij} plays exactly this role, as

$$\chi_{ij} = |\chi_{ij}|\mathrm{e}^{\mathrm{i}ij} \tag{11}$$

and the time component of the gauge field is given by a_0, already introduced. According to this the spinon pairing, Δ_{ij} has the gauge charge 2, and if it has non-zero expectation value, the gauge invariance is broken. Similarly the Bose condensation $\langle b_i\rangle$ also breaks the gauge invariance.

3. Angle-resolved photoemission spectra of the Mott insulator

Experimentally, angle-resolved photoemission spectroscopy (ARPES) became a powerful tool for investigating the issue of SCS because it gives information on the electron Green's function directly as a function of the momentum and energy. Recently Kim *et al* reported ARPES for one-dimensional $SrCuO_2$ [8], which is a typical example of a half-filled Mott insulator. In the spectra, two peak structures with different dispersions are observed, and Kim *et al* interpreted the results as the dispersions of the spinon and holon and hence the first experimental evidence for a SCS in 1D. Theoretically, the electron spectral function has been studied by several authors [9, 10].

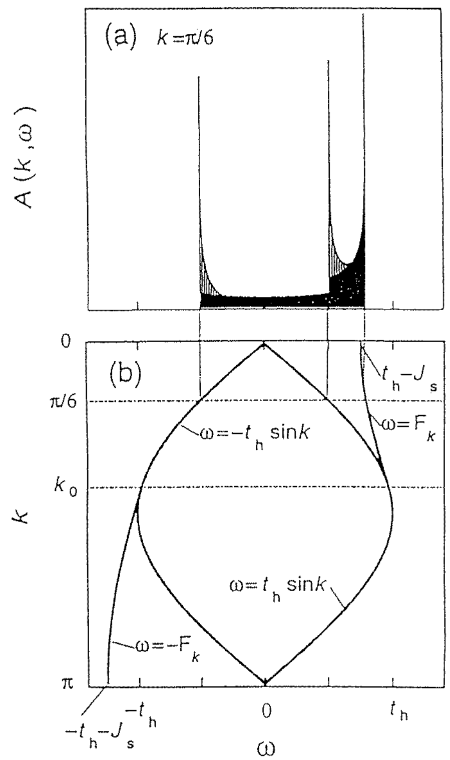

Figure 1. (a) The spectral function $A(k, \omega)$ with $k = \pi/6$. The simple slave-boson decomposition gives the fully shaded line shape. The partially shaded singularities are due to the non-local phase-string interactions. (b) The locations of the singularities are plotted in the ω–k plane.

We give here the simpler and physically transparent derivation of the spectrum in terms of the gauge theoretical picture [11]. We have employed the slave-boson formalism given in section 2, and the RVB mean-field Hamiltonian of the t–J model [5–7] is

$$H = -\frac{t_h}{2}\sum_{\langle i,j\rangle} b_i^\dagger b_j + \frac{J_s}{2}\sum_{\langle i,j\rangle,\sigma} f_{i,\sigma}^\dagger f_{j,\sigma} \tag{12}$$

where the spinons and holons are decoupled from the dispersions of the order of t and J, respectively. The electron Green's function, which is observed in ARPES, is given in real space-time by the product of those of the spinon and holon as [7]

$$G(r,t) = G_{\text{spinon}}(r,t)G_{\text{holon}}(-r,-t).$$

The Fourier transform $G(k,\omega)$ is given by the convolution, and the spectral function

$$A(k,\omega) = -\pi^{-1}\,\mathrm{Im}\,G(k,\omega)$$

is shown by the fully shaded line shape in figure 1.

The square-root singularity appears at the band edge of the holon and the energy dispersion of the spinon gives the F_k with the bandwidth $2J_s$. The cut-off energy has the dispersion t_h, but the peak structure with such a dispersion is missing here.

What is needed additionally to reproduce the correct behaviour is the interaction between spinons and holons, which is actually a gauge field, i.e., the non-local *phase string* connecting these particles [12]. After making a unitary transformation to take care of the sign change of the wavefunction, the electron operator is given by

$$\tilde{c}_{i\sigma} = f_{i\sigma} b_i^\dagger \mathrm{e}^{\mathrm{i}(\theta_i^h + \theta_i^s)} \tag{13}$$

with

$$\theta_i^h = \mp\frac{\pi}{2}\sum_{l>i} b_l^\dagger b_l \qquad \theta_i^s = \pm\frac{\pi}{2}\sum_{l>i}(f_{l\uparrow}^\dagger f_{l\uparrow} + f_{l\downarrow}^\dagger f_{l\downarrow} - 1).$$

Here, the spinon and the holon are described by the *free fermions* $f_{i\sigma}$ and b_i, respectively, but the original spinon and holon strongly interact with each other via non-local phase string. Now the system is at half-filling, and the phase-string term of the holon makes no contribution, but that of the spinon makes the long-distance behaviour of the spinon Green's function change from $G_{\text{spinon}}(x,t) \sim \mathrm{e}^{\pm \mathrm{i}k_s^F x}/(x \pm v_s t)$ to

$$G_{\text{spinon}}(x,t) = \langle f_{x,\sigma}^\dagger \mathrm{e}^{-\mathrm{i}\theta_x^s} \mathrm{e}^{\mathrm{i}\theta_0^s} f_{0,\sigma}\rangle \sim \frac{\mathrm{e}^{\pm \mathrm{i}k_s^F x}}{\sqrt{x \pm v_s t}} \tag{14}$$

where the Fermi velocity of the spinon

$$v_s = \mathrm{d}E_{k=k_s^F}^{\text{spinon}}/\mathrm{d}k = J_s.$$

The bosonization technique gives this asymptotic behaviour under the constraint $\langle f_{l\uparrow}^\dagger f_{l\uparrow} + f_{l\downarrow}^\dagger f_{l\downarrow}\rangle = 1$ [12].

Taking this asymptotics into account, the spectral function $A(k,\omega)$ is given by

$$\begin{aligned} A(k,\omega) &\sim \int \mathrm{d}x\ \mathrm{d}t\ \mathrm{d}k_h\ \mathrm{e}^{\mathrm{i}(\omega + E_{k_h}^{\text{holon}})t - \mathrm{i}(k + k_h \mp k_s^F)x} \frac{1}{\sqrt{x \pm v_s t}} \\ &\sim \int \mathrm{d}X\ \mathrm{d}k_h\ \mathrm{e}^{-\mathrm{i}(k + k_h \mp k_s^F)X} \frac{1}{\sqrt{X}} \delta(\omega + E_{k_h}^{\text{holon}} \mp v_s(k + k_h \mp k_F^s)). \end{aligned} \tag{15}$$

This result is valid when low-energy anti-spinons are created below the Fermi points—that is, when $\Delta k = k_h + k \mp k_F^s$ and $\Delta\omega = \omega + E_{-k\pm k_s^F}^{\text{holon}}$ (>0) are close to zero. The constraint originating from the δ-function is expanded in terms of Δk:

$$\begin{aligned} &\omega + E_{k_h}^{\text{holon}} \mp v_s(k + k_h \mp k_F^s) \\ &\quad = \Delta\omega \pm (t_h\cos k - J_s)\Delta k \mp \frac{1}{2}t_h \sin k(\Delta k)^2 + \mathrm{O}((\Delta k)^3) = 0. \end{aligned}$$

For $k \neq k_0 = \cos^{-1}(J_s/t_h)$, $\Delta\omega$ is proportional to Δk. So, the δ-function gives no singularity and we obtain

$$A(k,\omega) \sim \int \mathrm{d}X\ \frac{\mathrm{e}^{-\mathrm{i}\Delta k X}}{\sqrt{X}} \sim (\Delta k)^{-1/2} \propto (\Delta\omega)^{-1/2}. \tag{16}$$

Then the square-root singularities are restored as shown by the partially shaded line shape in figure 1(a). Summarizing, we found that the ARPES of $SrCuO_2$ show the existence of the spinon and holon and the non-local phase string between them.

4. SU(2) theory for underdoped cuprates

The mean-field picture of the phase diagram for high-T_c cuprates is the following. In the plane of the hole concentration x and the temperature T, we have four phases [6, 7]. The uniform RVB state corresponds to the state with non-zero χ_{ij}, but the spinon pairing Δ_{ij} and the Bose condensation $\langle b_i \rangle$ are absent. This uniform RVB state is unstable against the two instabilities. One is the spinon pairing as in the case of BCS theory, and the other is the holon condensation. The spin-gap state is characterized by the appearance of the spinon pairing to the uniform RVB state, which is considered to be realized in the underdoped region. The holon condensation, on the other hand, makes the system a Fermi liquid. This appears in the overdoped region. When both of these appear, the system is a superconductor. Theoretical studies of the anomalous physical properties in the normal state have been extensively carried out for the uniform RVB state, which corresponds to the optimally doped region [7]. However, if one tries to approach the underdoped spin-gap state, one encounters several difficulties summarized in reference [13]. These difficulties originate from the fact that the AF ordering cannot be described by the RVB order parameters, which represent singlet formation. This means that we need more and more fluctuation of U_{ij} in equation (7) when approaching $x = 0$. To search for the important low-energy fluctuations, we are led by symmetry considerations. It is known that at $x = 0$ the system has an SU(2) symmetry exchanging the particle and hole [14]. Explicitly, the SU(2) gauge transformation is defined as the rotation of the spinor ψ_i in terms of an SU(2) matrix g_i:

$$\begin{aligned} &\psi_i \to \tilde{\psi}_i = g_i^\dagger \psi_i \\ &U_{ij} \to \tilde{U}_{ij} = g_i^\dagger U_{ij} g_j. \end{aligned} \tag{17}$$

In equation (6) the Lagrangian L_F for the fermions is invariant with respect to this gauge transformation, but the holon contribution L_B is not. Then, away from the half-filling, U and $\tilde{U}$ are different configurations physically. However, the energy difference is expected to be small when x is small, and we have to include the fluctuation corresponding to the SU(2) rotation, equation (17). This has been done and the result is the SU(2) theory whose action is given by

$$\begin{aligned} \tilde{L} = \frac{\tilde{J}}{2} \sum_{\langle ij \rangle} \mathrm{Tr}\left[U_{ij}^\dagger U_{ij}\right] + \frac{1}{2} \sum_{i,j,\alpha} \psi_{\alpha i}^\dagger (\partial_\tau \delta_{ij} + \tilde{J} U_{ij}) \psi_{\alpha j} + \sum_{i\ell} a_{0i}^\ell \left(\frac{1}{2} \psi_{\alpha i}^\dagger \tau^\ell \psi_{\alpha i} + h_i^\dagger \tau^\ell h_i \right) \\ + \sum_{ij} h_i^\dagger ((\partial_\tau - \mu)\delta_{ij} + \tilde{t} U_{ij}) h_j. \end{aligned} \tag{18}$$

Corresponding to the rotation of the spinon in equation (17), the holon b_i is rotated to give the two-component holon h_i as follows:

$$h_i = g_i \begin{bmatrix} b_i \\ 0 \end{bmatrix}. \tag{19}$$

In terms of this SU(2) formulation, the mean-field phase diagram and the fluctuation around it has been clarified [13]. Here we focus in the next section on the problem of the confinement–deconfinement of the gauge field for the underdoped cuprates.

5. Confinement–deconfinement of the gauge field

In the gauge theoretical formulation, the spinon and holon have the quantum numbers $(Q, S) = (0, 1/2)$ and $(Q, S) = (e, 0)$, respectively. These quantum numbers are highly non-trivial, and in all of the conventional states the elementary excitations can be regarded as composite particles of the spinons and holons. For example, the triplet spin excitation in the Néel state is the bound state of two spinons, the electron in the Fermi-liquid state is that of a spinon and a holon, and the bipolaron in the doped two-leg ladder is the two-holon bound state. This means that the Fermi-liquid state, (doped) VBS state and the Néel state belong to the confining phase of the gauge field. This is incorporated in the gauge theoretical formulation as follows.

(1) The Fermi liquid is obtained when the holon is Bose condensed, where the gauge field has mass due to the Higgs phenomenon, and the Higgs phase is identical to the confining one.

(2) In the undoped case, the Heisenberg model can be mapped to the strong-coupling limit of the SU(2) lattice gauge theory [14]. The confining phase of the gauge field is usually accompanied with chiral symmetry breaking, which is equivalent to the AFLRO in the present case.

Hence all of the known conventional states are described at least qualitatively in terms of the gauge theoretical formulation with the confining gauge field. When the gauge field is deconfining, the new state of matter, i.e., the RVB state, is realized. We clarify the relevant physical quantity for confinement–deconfinement below.

In terms of the SU(2) formulation in section 4, the spin-gap state is described as the staggered flux state where the original SU(2) symmetry is broken down to U(1) [13]. In this case, the Higgs field is the particle–particle and/or particle–hole pair of the fermions, which belongs to the adjoint representation of the SU(2) symmetry. In the gauge theoretical language, the gauge charge of this Higgs field is not fundamental, and the Higgs phase and the confining phase are distinct phases. These two are distinguished by the confinement–deconfinement of the gauge field, which is the subject of the present section.

In the staggered flux state, the effective action describing the holons $h_i = {}^{\mathrm{t}}[b_{1i}, b_{2i}]$ in the underdoped spin-gap region is given by [13]

$$S = \int \mathrm{d}\tau \int \mathrm{d}r\ h^\dagger(r,\tau)\left[\partial_\tau + \mathrm{i}a_0^3\tau_3 + \mathrm{i}A_0 + \frac{1}{2m}(-\mathrm{i}\boldsymbol{\nabla} + \boldsymbol{a}^3\tau_3 + \boldsymbol{A})^2 - \mu\right]h(r,\tau) + \sum_{q,\omega} a_\mu^3(q,\omega)\Pi_{S\mu\nu}(q,\omega)a^3\nu(-q,-\omega) \tag{20}$$

where the spinons have been integrated over to give the polarization function Π_S in the effective action of the gauge field. Because the gauge symmetry is broken from SU(2) to U(1) in the staggered flux state, only the a^3 gauge field remains massless. Note also that the Ioffe–Larkin composition rule [15] no longer applies because the external vector potential A_μ is coupled to h_i by the unit matrix and not by τ_3. Then the conductivity is determined by that of the holon system. Thus in the limit of small q and ω the leading-order contributions of the holons to the effective action for the gauge fields A_μ and a_μ^3 are given as

$$\sum_{q,\omega} \sigma_{\mathrm{dc}}|\omega|(a_\perp^3(q,\omega)a_\perp^3(-q,-\omega) + A_\perp(q,\omega)A_\perp(-q,-\omega)) \tag{21}$$

where $a_\perp^3$ and $A_\perp$ are the transverse components. The contribution from the spinons Π_S, on the other hand, is small compared with equation (21) in the limit of small q, ω because of the d-wave-type gap in the spinon spectrum. Then the strength of the dissipation for the gauge field is determined by the dc conductivity (conductance) $\sigma_{\rm dc} = R_{\rm 2D}^{-1}$ of the system. The phase diagram of the U(1) gauge field with dissipation has been studied, and it was found that when $\sigma_{\rm dc}$ is larger than a critical value σ_c of the order of the quantum conductance e^2/h, the gauge field is deconfining, while it is confining otherwise [16]. On the other hand it is known also that the superconductor–insulator transition in the 2D Josephson network model is controlled by the conductance, and the criterion is similar to that given above. As discussed recently, the localized vacancies in the non-linear sigma model give the topological disorder (the random Berry phase term) which enhances the classical nature of the staggered magnetization and leads to the AFLRO [17]. Then we see the following correspondence:

(case A) spin–charge separation—metallic transport and superconductivity—the RVB state (the spin liquid); and

(case B) spin–charge confinement—localization and insulator behaviour—the Néel state (the spin solid).

According to this scenario, the ladder systems belong to case B before applying pressure. On increasing the pressure, the anisotropy of the resistivity decreases within the ladder plane, and eventually becomes superconducting when $R_{\rm 2D} = \sqrt{R_a R_b}$ becomes of the order of h/e^2 [18]. This suggests that the system turns into case A and the superconductivity is two dimensional. Note that the bipolaron will be dissociated into two charge-e holons because the gauge field is not confining any longer. There should be a quantum critical point between these two classes, as recently discussed by Fukuyama [19], which needs further investigation.

6. The Kondo effect in high-T_c cuprates

We now turn to the issue of the experimental evidence for the new state of matter. We propose that the effects of the non-magnetic impurities, e.g., Zn, replacing Cu atoms in the conducting layers, provide evidence for the spin–charge separation. These non-magnetic impurities induce the following anomalous properties.

(a) The formation of the magnetic moments due to Zn has been revealed by magnetic susceptibility [20], NMR [21], μSR [22] and EPR [23]; their magnitudes are roughly proportional to the Zn concentration and an almost full moment appears in the underdoped region, i.e., $0.8\mu_B$ for $La_{1.85}Sr_{0.15}Cu_{1-z}Zn_zO_4$ [24] and $0.86\mu_B$ for $YBa_2(Cu_{1-z}Zn_z)_3O_{6.64}$ [21], per Zn ion. The magnitude of the induced local moments is strongly dependent on the hole concentration x and becomes smaller or even vanishes as x increases [25].

(b) The residual resistivity can be described in terms of the classical expression in terms of the Boltzmann transport theory [25]. That is,

$$\rho_{\rm res} = 4(\hbar/e^2)(n_{\rm imp}/n)\sin^2\delta_0 \tag{22}$$

which is independent of the effective mass of the carriers and is determined only by the phase shift δ_0, the impurity concentration $n_{\rm imp}$ and the carrier density n. It has been found that Zn acts as a strong scatterer at the unitary limit, i.e., $\delta_0 = \pi/2$. The interesting thing is that the number of carriers n is the hole concentration x in the underdoped region, and rather rapidly crosses over to the electron number $1 - x$ in the overdoped region.

I now discuss how these anomalous features can be understood in terms of the gauge theory [26]. The transport and magnetic properties are determined in different ways in terms of the spinons and holons. The magnetic properties are dominated by the spinons, and the formation of the local spin moment has been attributed to the spin-gap formation in the underdoped region. Due to the linear density of states, the spinons cannot screen the local moment induced by the non-magnetic impurity, while the Kondo screening occurs in the optimally or overdoped region without the spin gap. Let us turn to the residual resistivity. The residual resistivity is determined by the phase shift for the spinons and holons. In the optimally and overdoped region, the usual argument in terms of the Friedel sum rule and Kondo effect applied to the spinons, while the holon phase shift is 0 or π because there is no spin label for holons. Then the residual resistivity is predicted to be $\rho_{\mathrm{res}} = 4(\hbar/e^2)n_{\mathrm{imp}}/(1-x)$ in agreement with experiments. In the underdoped region the situation is more anomalous because the number of carriers n in equation (8) is x, which contradicts the Fermi-liquid picture. Furthermore, ρ_{res} in the underdoped region is also inconsistent with the small-hole-pocket picture for the Néel state.

An SU(2) theory discussed in section 2 [13] resolves several difficulties in the U(1) formulation, and describes the smooth crossover from the small pockets to the large Fermi surface. A feature of this theory is that the holon is the two-component SU(2) doublet as in equation (2), and the SU(2) constraint is given by

$$\sum_{\sigma} f_{i\sigma}^{\dagger} f_{i\sigma} + b_{1i}^{\dagger} b_{1i} - b_{2i}^{\dagger} b_{2i} = 1. \tag{23}$$

Then the local moment formation means that $\Delta \sum_{\sigma} f_{i\sigma}^{\dagger} f_{i\sigma} = 1$ when we consider the large sphere including the Zn atom. This condition together with the neutrality condition determines the holon phase shift uniquely as $\delta_{b_2} = -\delta_{b_1} = \pi/2$, which gives the result $\rho_{\mathrm{res}} = 4(\hbar/e^2)n_{\mathrm{imp}}/x$ in agreement with experiments [25]. Hence we believe that the SU(2) formulation is essential for explaining the residual resistivity in the underdoped region.

Summarizing this section, we have analysed the Kondo effect in high-T_c cuprates on the basis of the spin–charge-separated state. The change of the phase shifts δ^{holon} and δ^{spinon} for holons and spinons due to the Kondo screening together with the crossover from SU(2) to U(1) theory explains the change of the residual resistivity from

$$\rho_{\mathrm{res}} = \frac{4\hbar}{e^2}\frac{n_{\mathrm{imp}}}{x}$$

to

$$\rho_{\mathrm{res}} = \frac{4\hbar}{e^2}\frac{n_{\mathrm{imp}}}{1-x}$$

as the hole concentration increases and the local moment disappears. Lastly we comment on the bipolaronic model for the underdoped cuprates. In this model the charge of the carrier is $2e$ and $n = x/2$ in the underdoped region. This gives

$$\rho_{\mathrm{res}} = \frac{2\hbar}{e^2}\frac{n_{\mathrm{imp}}}{x}$$

which is half of that expected on the basis of the above. Thus the experiments support the existence of a carrier not with charge $2e$ but with charge e.

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